The Role of Higher Twist and Positivity Constraints in Determining Polarized Parton Densities

E. Leader (London), A. Sidorov (Dubna), D. Stamenov (Sofia)

OUTLINE

- New QCD fits to the inclusive polarized DIS data
 - two sets of **polarized** PD (in both the MS and the JET schemes)

JLab Hall A neutron data very recent COMPASS data on A₁^d

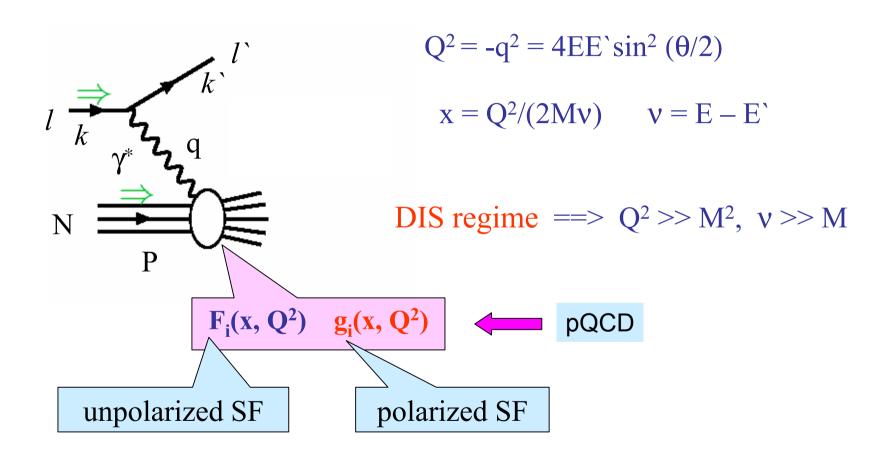


included in the analysis

- Role of **higher twist** in determining polarized PD
- Factorization scheme dependence of the results
- Impact of positivity constraints on polarized PD
- Summary

Inclusive DIS

one of the best tools to study the structure of **nucleon**



As in the unpolarized case the main goal is:

- to test QCD
- to extract from the DIS data the **polarized** PD

$$\Delta q(x,Q^{2}) = q_{+}(x,Q^{2}) - q_{-}(x,Q^{2})$$

$$\Delta \overline{q}(x,Q^{2}) = \overline{q}_{+}(x,Q^{2}) - \overline{q}_{-}(x,Q^{2})$$

$$\Delta G(x,Q^{2}) = G_{+}(x,Q^{2}) - G_{-}(x,Q^{2})$$

where "+" and "-" denote the helicity of the parton, along or opposite to the helicity of the parent nucleon, respectively.

The knowledge of the polarized PD will help us:

- to make predictions for other processes like polarized hadron-hadron reactions, etc.
- more generally, to answer the question how the helicity of the nucleon is divided up among its constituents:

$$S_z = 1/2 = 1/2 \Delta\Sigma(Q^2) + \Delta G(Q^2) + L_z(Q^2)$$

$$\Delta \Sigma = \Delta u + \Delta \overline{u} + \Delta d + \Delta \overline{d} + \Delta s + \Delta \overline{s}$$

the parton polarizations Δq_a and ΔG are the first moments

$$\Delta q_a(Q^2) = \int_0^1 dx \Delta q_a(x, Q^2) \ \Delta G(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

of the helicity densities: $\Delta u(x,Q^2), \Delta \overline{u}(x,Q^2), \dots, \Delta G(x,Q^2)$

DIS Cross Section Asymmetries

Measured quantities

$$A_{\parallel} = \frac{d\sigma^{\downarrow \uparrow} - d\sigma^{\uparrow \uparrow}}{d\sigma^{\downarrow \uparrow} + d\sigma^{\uparrow \uparrow}}, \qquad A_{\perp} = \frac{d\sigma^{\downarrow \Rightarrow} - d\sigma^{\uparrow \Rightarrow}}{d\sigma^{\downarrow \Rightarrow} + d\sigma^{\uparrow \Rightarrow}}$$

$$A_{\perp} = \frac{d\sigma^{\downarrow \Rightarrow} - d\sigma^{\uparrow \Rightarrow}}{d\sigma^{\downarrow \Rightarrow} + d\sigma^{\uparrow \Rightarrow}}$$

$$(A_{\parallel},A_{\perp}) \Rightarrow (A_{\perp},A_{2}) \Rightarrow (g_{\perp},g_{2})$$

where A_1 , A_2 are the virtual photon-nucleon asymmetries.

At present, A_{\parallel} is much better measured than A_{\perp}

If A_{\parallel} and A_{\perp} are measured

$$\Rightarrow g_1/F_1$$

If only A_{\parallel} is measured

$$\Rightarrow \frac{A_{\parallel}^{N}}{D} \approx (1 + \gamma^{2}) \frac{g_{1}}{F_{1}}$$

$$\gamma^2 = 4M_N^2 x^2 / Q^2$$
 - kinematic factor

NB. γ cannot be neglected in the SLAC, **HERMES** and **JLab** kinematic regions

$$\mathbf{A}^d$$

CERN EMC -
$$A_1^p$$
 SMC - A_1^p , A_1^d COMPASS - A_1^d

188 exp. p.

DESY HERMES -
$$\frac{g_1^p}{F_1^p}$$
, A_1^n
SLAC E142, E154 - A_1^n E143, E155 - $\frac{g_1^p}{F_1^p}$, $\frac{g_1^d}{F_1^d}$

200 exp. p.

$$A_1^n$$

$$\frac{g_1^p}{F^p}, \frac{g_1^a}{F^a}$$

JLab Hall A -
$$\frac{g_1^n}{F_n}$$

The data on A_1 are really the experimental values of the quantity

$$\frac{A_{||}^{N}}{D} = (1 + \gamma^{2}) \frac{g_{1}^{N}}{F_{1}^{N}} + (\eta - \gamma) A_{2}^{N}$$
$$= A_{1}^{N} + \eta A_{2}^{N}$$

 $\gamma \approx \eta$ and A_2 small

very well approximated with $(1+\gamma^2)\frac{g_1^N}{F^N}$ even when $\gamma(\eta)$ can not be neglected

$$(1+\gamma^2)\frac{g_1^N}{F_1^N}$$

• An important difference between the kinematic regions of the unpolarized and *polarized* data sets

A lot of the present data are at **moderate** Q^2 and W^2 :

$$Q^2 \approx 1 - 5 \, GeV^2$$
, $4 < W^2 < 10 \, GeV^2$

preasymptotic region

While in the determination of the PD in the unpolarized case we can cut the low Q² and W² data in order to eliminate the less known non-perturbative HT effects, it is impossible to perform such a procedure for the present data on the spin-dependent structure functions without loosing too much information.

$$O(1/Q^2)$$



HT corrections should be important in polarized DIS!

Theory

In QCD

$$g_1(x,Q^2) = g_1(x,Q^2)_{LT} + g_1(x,Q^2)_{HT}$$

$$g_1(x,Q^2)_{LT} = g_1(x,Q^2)_{pQCD}$$

$$g_1(x,Q^2)_{HT} = h(x,Q^2)/Q^2 + h^{TMC}(x,Q^2)/Q^2$$

target mass corrections which are calculable

J. Blumlein, A. Tkabladze

dynamical HT power corrections ($\tau = 3,4$)

In NLO pQCD

=> non-perturbative effects (model dependent)

$$g_{1}(x,Q^{2})_{pQCD} = \frac{1}{2} \sum_{q}^{N_{f}} e_{q}^{2} \left[(\Delta q + \Delta \overline{q}) \otimes (1 + \frac{\alpha_{s}(Q^{2})}{2\pi} \delta C_{q}) + \frac{\alpha_{s}(Q^{2})}{2\pi} \Delta G \otimes \frac{\delta C_{G}}{N_{f}} \right]$$

 δC_q , δC_G – Wilson coefficient functions

polarized PD evolve in Q²

according to NLO DGLAP eqs.

 N_f (=3) - a number of flavours

Test of QCD and determination of PPD

$$(\Delta q_i, \Delta \overline{q_i}, \Delta G)(x, Q_0^2; a_k) \xrightarrow{\text{pQCD}} (\Delta q_i, \Delta \overline{q_i}, \Delta G)(x, Q^2; a_k)$$

$$\text{DGLAP eqs.}$$

Input PD

a_k – free par.

$$g_1(x,Q^2;a_k)_{pQCD}$$

$$\chi^{2} = \sum_{i,j} \frac{\left[g_{1}(x_{i}, Q_{j}^{2})_{\exp} - g_{1}(x_{i}, Q_{j}^{2}; a_{k})_{pQCD}\right]^{2}}{\Delta g_{1}(x_{i}, Q_{j}^{2})_{\exp}^{2}}$$

$$\rightarrow a_k \pm \Delta a_k$$

Methods of analysis

Fit to g_1/F_1 data - g_1/F_1 fit => PD(g_1/F_1) or Set 1

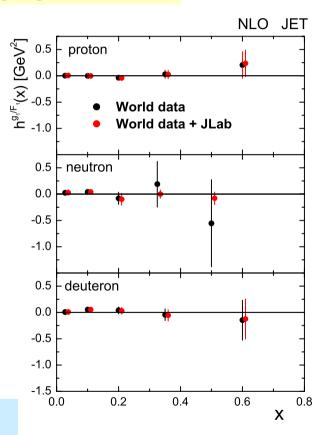
$$\left[\frac{g_1(x,Q^2)}{F_1(x,Q^2)}\right]_{\text{exp}} \iff \frac{g_1(x,Q^2)_{LT}}{F_1(x,Q^2)_{LT}} + \frac{h^{g_1/F_1}(x)}{Q^2}$$

$$(g_1)_{QCD} = (g_1)_{LT} + (g_1)_{HT}$$

$$(F_1)_{QCD} = (F_1)_{LT} + (F_1)_{HT}$$

$$\Rightarrow h^{g_1/F_1} \approx 0 \Rightarrow \frac{(g_1)_{HT}}{(g_1)_{LT}} \approx \frac{(F_1)_{HT}}{(F_1)_{LT}}$$

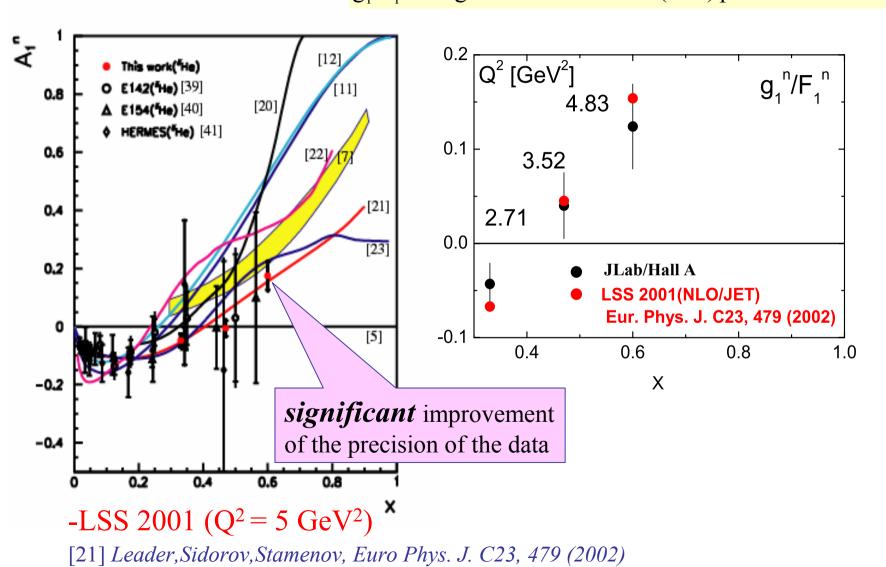
The HT corrections to g_1 and F_1 approximately compensate each other in the ratio g_1/F_1 and the PPD extracted this way are less sensitive to HT effects



LSS: EPJ C23 (2002) 479 hep-ph/0309048

JLab/Hall A: PRL 92 (2004) 012004

Our predictions for the JLAB experimental values of g_1^n/F_1^n using the LSS'2001 NLO(JET) polarized PD



Fit to g_1/F_1 data - Gluck et al. (GRSV); Leader et al. (LSS)

$$\left[\frac{g_{1}(x,Q^{2})}{F_{1}(x,Q^{2})}\right]_{\text{exp}} \stackrel{\chi^{2}}{\longleftrightarrow} \frac{g_{1}(x,Q^{2})_{LT}}{F_{1}(x,Q^{2})_{LT}}; \qquad \chi^{2}_{dof} = 0.884$$

$$\begin{bmatrix} \frac{g_1(x,Q^2)}{F_1(x,Q^2)} \end{bmatrix}_{\text{exp}} \iff \frac{g_1(x,Q^2)_{LT}}{F_1(x,Q^2)_{LT}} + \frac{h^{g_1/F_1}(x)}{Q^2} \qquad \qquad \qquad \qquad h^{g_1/F_1}(x) \approx 0$$

Fit to g_1 data - SMC; Blumlein, Bottcher (BB); AAC

$$g_1(x,Q^2)_{\text{exp}} = \left[\frac{g_1(x,Q^2)}{F_1(x,Q^2)}\right]_{\text{exp}} F_1(x,Q^2)_{\text{exp}} \stackrel{\chi^2}{\Leftarrow \Rightarrow} g_1(x,Q^2)_{LT}$$

$$(F_2)_{\text{exp}}, R_{\text{exp}}$$

$$g_1(x,Q^2)_{\text{exp}} \stackrel{\chi^2}{\underset{212.5}{\Leftarrow}} g_1(x,Q^2)_{LT}$$

$$g_{1}(x,Q^{2})_{\exp} \stackrel{\chi^{2}}{\Longleftrightarrow} g_{1}(x,Q^{2})_{LT} = g_{1}(x,Q^{2})_{\exp} \stackrel{\chi^{2}}{\Longleftrightarrow} g_{1}(x,Q^{2})_{LT} + h^{g_{1}}(x)/Q^{2}; \quad \chi^{2}_{dof} = 0.886$$

important

Fit to g_1 data - g_1 +HT fit => PD(g_1 +HT) or Set 2

$$\left[\frac{g_1(x,Q^2)}{F_1(x,Q^2)}\right]_{\text{exp}} F_1(x,Q^2)_{\text{exp}} = g_1(x,Q^2)_{\text{exp}} \iff g_1(x,Q^2)_{LT} + h^{g_1}(x)/Q^2$$

 F_2^{NMC} , R_{1008} (SLAC)

in model independent way

HT corrections to g_1 cannot be compensated because the HT corrections to F₁(F₂ and R) are absorbed in the phenomenological parametrizations of the data on F₂ and R.

Input PD
$$\Delta f_i(x, Q_0^2) = A_i x^{\alpha_i} f_i^{MRST}(x, Q_0^2)$$
 $Q_0^2 = 1 \text{ GeV}^2, A_i, \alpha_i - \text{ free par.}$

 $h^p(x_i), h^n(x_i) - 10$ parameters (i = 1, 2, ... 5) to be determined from a fit to the data

8-2(SR) = 6 par. associated with PD; positivity bounds imposed by MRST'02 unpol. PD

$$g_A = (\Delta u + \Delta u)(Q^2) - (\Delta d + \Delta d)(Q^2) = F - D = 1.2670 \pm 0.0035$$

$$a_8 = (\Delta u + \Delta u)(Q^2) + (\Delta d + \Delta d)(Q^2) - 2(\Delta s + \Delta s)(Q^2) = 3F - D = 0.585 \pm 0.025$$

Flavor symmetric sea convention: $\Delta u_{sea} = \Delta u = \Delta d_{sea} = \Delta d = \Delta s = \Delta s$

SR for n=1 moments of PD

$$g_A = (\Delta u + \Delta \overline{u})(Q^2) - (\Delta d + \Delta \overline{d})(Q^2) = 1.2670 \pm .0035$$
 (1)

$$a_8 = (\Delta u + \Delta \bar{u})(Q^2) + (\Delta d + \Delta \bar{d})(Q^2)$$

$$-2(\Delta s + \Delta \bar{s})(Q^2) = 3F - D = 0.585 \pm 0.025$$
 (2)

The sum rule (1) reflects the isospin SU(2) symmetry, whereas the relation (2) is a consequence of the SU(3) flavour symmetry treatment of the hyperon β -decays.

While isospin symmetry is not in doubt, there is some question about the accuracy of assuming $SU(3)_f$ symmetry in analyzing hyperon β -decays. The results of the recent KTeV experiment at Fermilab on the β -decay of Ξ^0 , $\Xi^0 \to \Sigma^+ e \overline{\nu}$, however, are all *consistent* with *exact* $SU(3)_f$ symmetry. Taking into account the experimental uncertainties one finds that $SU(3)_f$ breaking is at most of order 20%.

KTeV experiment Fermilab

$$\Xi^0 \to \Sigma^+ e \overline{\nu}$$

β-decay

 $SU(3)_f$ prediction for the form factor ratio g_1/f_1

$$\frac{g_1}{f_1} = g_A = 1.2670 \pm .0035$$

Experimental result

$$\frac{g_1}{f_1} = 1.32^{+0.21}_{-0.17} \pm 0.05$$

A good agreement with the *exact* SU(3)_f symmetry!

From exp. uncertainties



SU(3) breaking is at most of order **20%**

RESULTS OF ANALYSIS

- $(\Delta u + \Delta \overline{u}), (\Delta d + \Delta \overline{d})$ well determined
- $(\Delta s + \Delta s)$ reasonably well determined and negative if accept for a_8 its SU(3) symmetric value $a_8 = 3F-D = 0.58$
- ΔG not well constrained

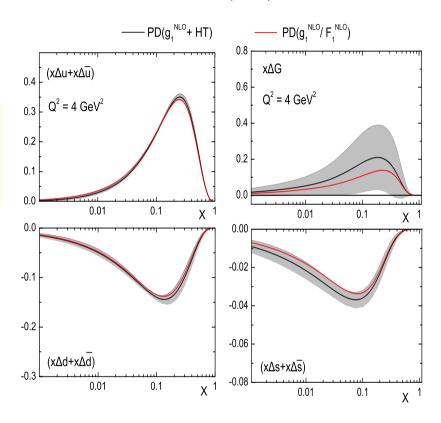
$$PD(g_1^{NLO} + HT) \Leftrightarrow PD(g_1^{NLO} / F_1^{NLO})$$

$$\chi^2_{DF,NLO} = 0.872 \Leftrightarrow \chi^2_{DF,NLO} = 0.874$$



In g_1 data fit HT corrections are important!

$NLO(\overline{MS})$



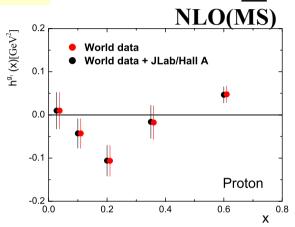
The two sets of polarized PD are very close to each other, especially for u and d quarks.

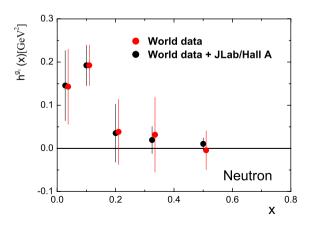
Higher twist effects

- The size of HT coorections to g₁ is NOT negligible
- The shape of HT depends on the target
- Thanks to the very precise JLab Hall A data the higher twist corrections for the neutron target are now much better determined at large x.

$$\int_{0}^{1} dx \, h^{g_1}(x) = \frac{4}{9} M^2 (d_2 + f_2)$$
HT (\tau=3) HT (\tau=4)

Our result is in agreement with the instanton model predictions (Balla et al., NP B510, 327, 1998) but disagrees with the renormalon calculations (Stein, NP 79, 567, 1999).





Main goal

To extract correctly PPD including the data in the preasymptotic region (Q^2 : 1 – 5 GeV², W^2 > 4 GeV²)

 $g_1(p,n,d)$

Mainly to study the HT effects.

The data in the **resonanse** region are also included

The analysis is performed

in Bjorken x-space

Data set

in n-space of the Nachtmann moments of g₁

 g_1^p

LT + HT approximations

NLO, $O(1/Q^2)$

NLO⊕SGR (soft gluon resummation)

$$O(1/Q^2) + O(1/Q^4)$$

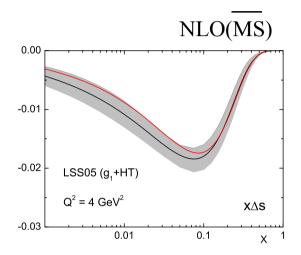
- Not easy to compare directly the results of the two analyses
- Is the quark-hadron duality satisfied in the polarised case?

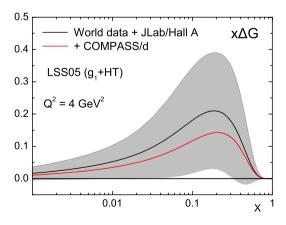
(A.Fantoni et al., hep-ph/0501180)

Effect of COMPASS A_1^d data (hep-ph/0501073) on polarized PD and HT

- The statistical accuracy at small x: 0.004 < x < 0.03 is **considerably** improved
- $\Delta u_v(x)$ and $\Delta d_v(x)$ do **NOT** change in the exp. region
- $x|\Delta s(x)|$ and $x \Delta G(x)$ decrease, but the corresponding curves lie within the error bands

LSS'05: hep-ph/0503140





COMPASS (high p_t hadron pairs with $Q^2 > 1 \text{ GeV}^2$) – hep-ex/0501056

for x=0.13, $Q^2=2 \text{ GeV}^2$

$$\Delta G/G = 0.06 \pm 0.31(\text{stat}) \pm 0.06(\text{sys}) \text{ at } < x_G > = 0.13 \pm 0.08$$

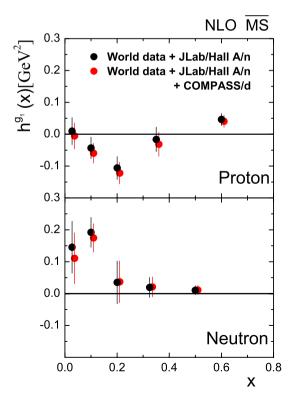
LSS'05 result

$$\Delta$$
G/G = 0.095 Set 2/NLO(MS)

G(x,Q2) is the NLO MRST'02 unpolarized gluon density

Effect of the COMPASS data on the HT values

- The new values are in good agreement with the old ones
- The COMPASS data are in the DIS region their effect on HT is **negligible**



Factorization scheme dependence

NLO polarized PD in MS and JET schemes

 In NLO QCD the valence quarks and gluons should be the same in both schemes, while

$$\Delta s(x,Q^2)_{JET} = \Delta s(x,Q^2)_{\overline{MS}} + \frac{\alpha_S}{2\pi} (1-x) \otimes \Delta G(x,Q^2)_{\overline{MS}}$$

n=1:
$$\Delta \Sigma_{JET} = \Delta \Sigma (Q^2)_{\overline{MS}} + 3 \frac{\alpha_S(Q^2)}{2\pi} \Delta G(Q^2)_{\overline{MS}}$$

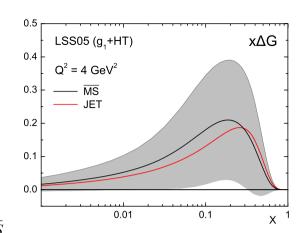
 $\Delta\Sigma_{\text{JFT}}$ is a **Q**² independent quantity

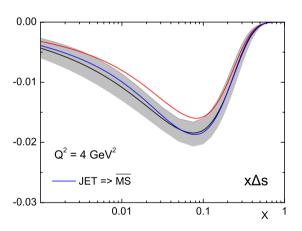
$$\Delta\Sigma_{\rm JET}({\rm DIS}) <=> \Delta\Sigma({\rm Q2}\sim\Lambda^2_{\rm QCD})$$

$$O^2 = 1 \text{ GeV}^2$$

CQM, chiral models

Fit	$\Delta\Sigma(Q^2)_{\overline{\rm MS}}$	$\Delta G(Q^2)_{JET}$	$\Delta\Sigma_{JET}$
LSS01	0.21± 0.10	0.68± 0.32	0.37± 0.07
LSS05	0.19 _± 0.06	0.29 _± 0.32	0.29 _± 0.08





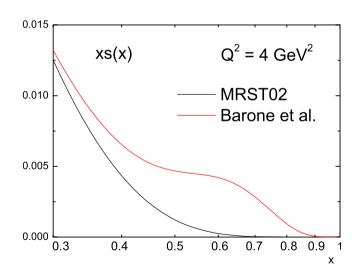
Our numerical results for PPD are in a good agreement with pQCD

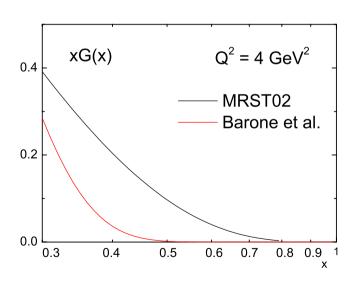
Impact of positivity constraints on polarized PD

LSS'01
$$\longrightarrow$$
 LSS'05 (Set 1)
$$|\Delta f(x)| \le f(x)_{Bar.} \qquad |\Delta f(x)| \le f(x)_{MRST02}$$

Bar.: Barone et al., EPJ C12 (2000) 243

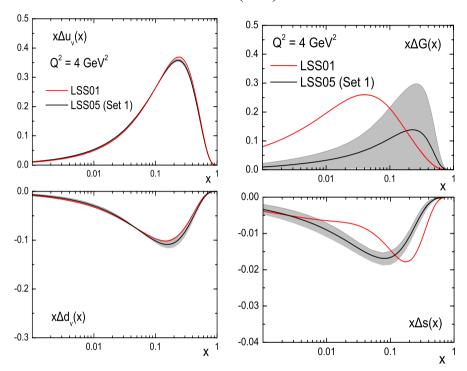
MRST02: EPJ C28 (2003) 455





At large x: $s(x)_{Bar} > s(x)_{MRST02}$ $G(x)_{Bar} < G(x)_{MRST02}$

$NLO(\overline{MS})$



Flavour symmetric sea convention:

$$\Delta u_{sea} = \Delta \overline{u} = \Delta d_{sea} = \Delta \overline{d} = \Delta s = \Delta \overline{s}$$

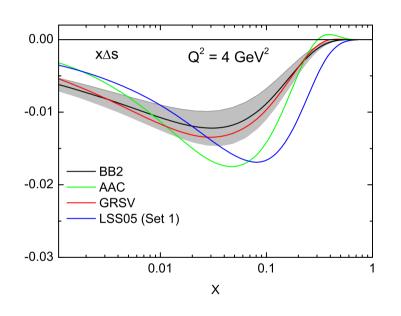
• Δu_v and Δd_v of the two sets are closed to each other

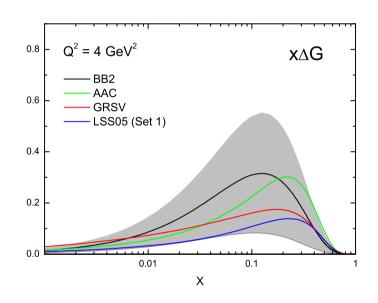
• Δ s and Δ G are **significantly** different

Δs and ΔG are weakly constrained from the data, especially for high x. That is why the role of positivity constraints is very important for their determination in this region.

NLO QCD PPD (MS) obtained by different groups

 $x\Delta s$ and $x\Delta G$ are **weakly** constrained from the present data on inclisive DIS





GRSV: Glück et al., hep-ph/0011215

BB: Blümlein, Böttcher, hep-ph/0203155

AAC: Goto et. al., hep-ph/0312112

LSS'05: Leader at al., hep-ph/0503140

 $x\Delta u_v$ and $x\Delta d_v$ well consistent

Impact of positivity constraints on $x\Delta s(x, Q^2)$

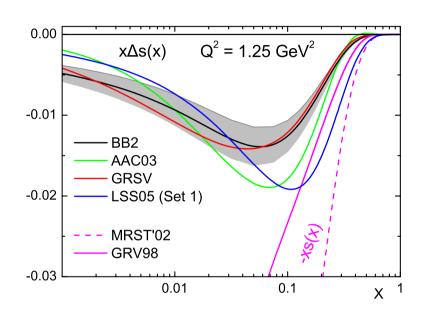
GRSV: Glück et al., hep-ph/0011215

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AAC: Goto et. al., hep-ph/0312112 LSS'05: Leader at al., hep-ph/0503140

$$|x\Delta f(x,Q_0^2)| \le xf(x,Q_0^2)_{GRV}$$

$$| x \Delta f(x, Q_0^2) |_{LSS} \le x f(x, Q_0^2)_{MRST02}$$

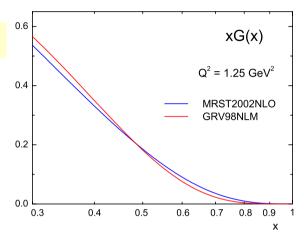


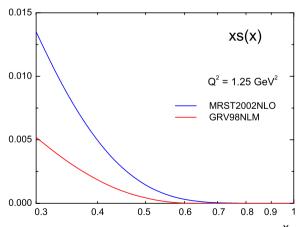
GRSV, BB and AAC have used the **GRV unpolarized** PD for constraining their PPD, while LSS have used those of **MRST'02**.

As a result, $x|\Delta s(x)|$ (LSS) for x > 0.1 is **larger** than the magnitude of the polarized strange sea densities obtained by the other groups.

Role of unpolarized PD in determining PPD at large x

- At large x the unpolarized GRV and MRST'02 **gluons** are practically **the same**, while $xs(x)_{GRV}$ is much smaller than that of MRST'02.
- For the adequate determination of $x\Delta s$ and $x\Delta G$ at large x, the role of the corresponding **unpolarized** PD is very important.
- Usually the sets of unpolarized PD are extracted from the data in the DIS region using cuts in Q² and W² chosen in order to minimize the higher twist effects.
- The latter have to be determined with good accuracy at large x in the **preasymptotic** (Q^2, W^2) region too.



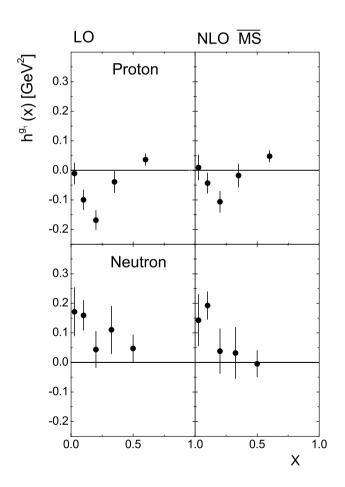


LO QCD approximation - NOT reasonable in the preasymptotic region

- $\alpha_s(Q^2)$ is large
- HT effects are large

Dependence of χ^2 on HT corrections

Fit	LO HT=0	NLO HT=0	LO+HT	NLO+HT
χ^2	249.8	212.5	153.8	149.8
DF	185-8	185-6	185-16	185-16
χ^2 /DF	1.41	1.19	0.910	0.886



$$\begin{bmatrix} \underline{g_1} \\ F_1 \end{bmatrix}_{\text{exp}} & \overset{\boldsymbol{\chi}_{DF}^2}{\Longleftrightarrow} & \underline{g_1^{LO}} \\ 0.92 & & & & \\ \hline
\end{bmatrix}$$

$$\chi_{DF}^2(NLO) = 0.87$$

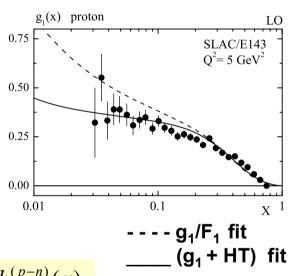
$$2xF_1^{LO} = F_2^{LO} \iff (q_a^{LO}, \bar{q}_a^{LO})$$

- at large Q²: $2x(F_1)_{exp} \approx (F_2)_{exp}$
- preasymt. region : $2x(F_1)_{exp} < (F_2)_{exp} (25-30\%)$
- E04-113, Semi-Sane exp. at JLab Hall C

$$\Delta \overline{u} - \Delta \overline{d} = \frac{1}{2} (\Delta q_3 - \Delta u_V + \Delta d_V)$$

In LO:
$$\Delta q_3(x,Q^2) = 6g_1^{(p-n)}(x,Q^2)_{\text{exp}}$$

In preas. region:
$$\Delta \widetilde{q}_3(x,Q^2) = 6 \left[g_1^{(p-n)}(x,Q^2)_{\exp} - \frac{h^{(p-n)}(x)}{Q^2} \right]$$



$$\frac{h^{(p-n)}(x)}{Q^2}]$$

If
$$x \in [0.1 - 0.4]$$
, $Q^2 = 2 \text{ GeV}^2$



HT contribution is about 24-34% (LSS'05)

SUMMARY

- Two sets of **polarized** PD in both the MS and the JET schemes are extracted from the world DIS data including the new **JLab** and **COMPASS** data in a **good agreement** with the pQCD predictions
- While the HT corrections to g_1 and F_1 compensate each other in g_1/F_1 , the HT(g_1) are **important** in the analysis of the g_1 data
- Impact of JLab data on PPD and HT ⇒ PPD unchanged, HT for a **neutron** target **much better determined** at high x
- Impact of COMPASS data on PPD $\Longrightarrow \Delta u_v$ and Δd_v unchanged, $|\Delta s|$ and ΔG decrease
- \triangle s and \triangle G are **not** well determined from the data the effect of the positivity conditions used to constrain them is **essential**, especially at high x
- A more precise determination of unpolarized PD in the preasymptotic region is very important